

# Package: SplitKnockoff (via r-universe)

August 23, 2024

**Type** Package

**Title** Split Knockoffs for Structural Sparsity

**Version** 1.1

**Date** 2022-02-20

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**Description** Split Knockoff is a data adaptive variable selection framework for controlling the (directional) false discovery rate (FDR) in structural sparsity, where variable selection on linear transformation of parameters is of concern. This proposed scheme relaxes the linear subspace constraint to its neighborhood, often known as variable splitting in optimization. Simulation experiments can be reproduced following the Vignette. We include data (both .mat and .csv format) and application with our method of Alzheimer's Disease study in this package. 'Split Knockoffs' is first defined in Cao et al. (2021) <[arXiv:2103.16159](https://arxiv.org/abs/2103.16159)>.

**URL** <https://github.com/wanghaoxue0/SplitKnockoff>

**BugReports** <https://github.com/wanghaoxue0/SplitKnockoff/issues>

**Depends** R (>= 3.5.0)

**Imports** glmnet, MASS, latex2exp, RSpectra, ggplot2, Matrix, stats,  
mvtnorm

**Suggests** knitr, rmarkdown

**Encoding** UTF-8

**VignetteBuilder** knitr

**NeedsCompilation** no

**RoxygenNote** 7.1.2

**License** MIT + file LICENSE

**Date/Publication** 2021-11-02 22:20:15 UTC

**Repository** <https://wanghaoxue0.r-universe.dev>

**RemoteUrl** <https://github.com/wanghaoxue0/splitknockoff>

**RemoteRef** HEAD

**RemoteSha** 259761699f55a1b4bf60fddc4c659263812d341f

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canonicalSVD	<i>singular value decomposition</i>
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### Description

Computes a reduced SVD without sign ambiguity

### Usage

canonicalSVD(X)

### Arguments

X            the input matrix

### Value

S  
U  
V

**Examples**

```

nu = 10
n = 350
m = 100
A_gamma <- rbind(matrix(0,n,m),-diag(m)/sqrt(nu))
svd.result = canonicalSVD(A_gamma)
S <- svd.result$S
S <- diag(S)
V <- svd.result$V

```

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hittingpoint	<i>hitting point calculator on a given path</i>
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**Description**

calculate the hitting time and the sign of respective variable in a path.

**Usage**

```
hittingpoint(coef, lambda_vec)
```

**Arguments**

coef	the path for one variable
lambda_vec	respective value of lambda in the path

**Value**

Z: the hitting time  
r: the sign of respective variable at the hitting time

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normc	<i>default normalization function for matrix</i>
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**Description**

normalize columns of a matrix.

**Usage**

```
normc(X)
```

**Arguments**

X	the input matrix
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**Value**

Y the output matrix

**Examples**

```
library(mvtnorm)
n = 350
p = 100
Sigma = matrix(0, p, p)
X <- rmvnorm(n, matrix(0, p, 1), Sigma)
X <- normc(X)
```

---

simu\_eval

*functions for evaluating simulations*

---

**Description**

calculate the FDR and Power for simulations.

**Usage**

```
simu_eval(gamma_true, result, r)
```

**Arguments**

gamma_true	true signal of gamma
result	the estimated support set of gamma
r	the estimated directional effect

**Value**

fdr: false discovery rate of the estimated support set

power: power of the estimated support set

---

simu_unit	<i>default unit for simulations</i>
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**Description**

the simulation unit for simulation experiments.

**Usage**

```
simu_unit(n, p, D, A, c, k, option)
```

**Arguments**

n	the sample size
p	the dimension of variables
D	the linear transform
A	SNR
c	feature correlation
k	number of nonnulls in beta
option	option for split knockoffs

**Value**

simu\_data: a structure contains the following elements

simu\_data\$fdr\_split: a vector recording fdr of split knockoffs w.r.t.nu

simu\_data\$power\_split: a vector recording power of split knockoffs w.r.t.nu

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sk.create	<i>generate split knockoff copies</i>
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**Description**

Give the variable splitting design matrix and response vector. It will also create a split knockoff copy if required.

**Usage**

```
sk.create(X, y, D, nu, option)
```

**Arguments**

X	the design matrix
y	the response vector
D	the linear transform
nu	the parameter for variable splitting
option	options for creating the Knockoff copy; option\$copy true : create a knockoff copy; option\$eta the choice of eta for creating the split knockoff copy

**Value**

A\_beta: the design matrix for beta after variable splitting

A\_gamma: the design matrix for gamma after variable splitting

tilde\_y: the response vector after variable splitting.

tilde\_A\_gamma: the knockoff copy of A\_beta; will be [] if option\$copy = false.

**Examples**

```

option <- array(data = NA, dim = length(data), dimnames = NULL)
option$q <- 0.2
option$eta <- 0.1
option$method <- 'knockoff'
option$normalize <- 'true'
option$lambda <- 10.^seq(0, -6, by=-0.01)
option$nu <- 10
option$copy <- 'true'
option$sign <- 'enabled'
option <- option[-1]
library(mvtnorm)
sigma <- 1
p <- 100
D <- diag(p)
m <- nrow(D)
n <- 350
nu = 10
c = 0.5
Sigma = matrix(0, p, p)
for( i in 1: p){
  for(j in 1: p){
    Sigma[i, j] <- c^(abs(i - j))
  }
}
X <- rmvnorm(n,matrix(0, p, 1), Sigma)
beta_true <- matrix(0, p, 1)
varepsilon <- rnorm(n) * sqrt(sigma)
y <- X %*% beta_true + varepsilon
creat.result <- sk.create(X, y, D, nu, option)
A_beta <- creat.result$A_beta
A_gamma <- creat.result$A_gamma
tilde_y <- creat.result$tilde_y

```

```
tilde_A_gamma <- creat.result$tilde_A_gamma
```

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sk.decompose	<i>make SVD as well as orthogonal complements</i>
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### Description

make SVD as well as orthogonal complements

### Usage

```
sk.decompose(X, randomize)
```

### Arguments

X	the input matrix
randomize	whether to randomize

### Value

U  
S  
V  
U\_perp : orthogonal complement for U

### Examples

```
library(mvtnorm)
n = 350
p = 100
Sigma = matrix(0, p, p)
X <- rmvnorm(n, matrix(0, p, 1), Sigma)
decompose.result <- sk.decompose(X)
U_perp <- decompose.result$U_perp
```

---

sk.filter

*Split Knockoff filter for structural sparsity*


---

### Description

the main function, Split Knockoff filter, for variable selection in structural sparsity problem.

### Usage

```
sk.filter(X, D, y, option)
```

### Arguments

X	the design matrix
D	the linear transform
y	the response vector
option	various options for split knockoff filter, the details will be specified in the example

### Value

results: a cell with the selected variable set in each cell w.r.t. nu.

Z: a cell with the feature significance Z in each cell w.r.t. nu.

t\_Z: a cell with the knockoff significance  $\tilde{Z}$  in each cell w.r.t. nu.

### Examples

```
option <- list(data = NA, dim = length(data), dimnames = NULL)

# the target (directional) FDR control
option$q <- 0.2

# choice on threshold, the other choice is 'knockoff+'
option$method <- 'knockoff'

# degree of separation between original design and its split knockoff copy
# in the range of [0, 2], the less the more separated
option$eta <- 0.1

# whether to normalize the dataset
option$normalize <- 'true'

# choice on the set of regularization parameters for split LASSO path
option$lambda <- 10.^seq(0, -6, by=-0.01)

# choice of nu for split knockoffs
option$nu <- 10
```



```

# choice on whether to estimate the directional effect, 'disabled'/'enabled'
option$sign <- 'enabled'

option <- option[-1]

# Settings on simulation parameters
k <- 20 # sparsity level
A <- 1 # magnitude
n <- 350 # sample size
p <- 100 # dimension of variables
c <- 0.5 # feature correlation
sigma <- 1 # noise level
# generate D
D <- diag(p)
m <- nrow(D)
# generate X
Sigma = matrix(0, p, p)
for( i in 1: p){
  for(j in 1: p){
    Sigma[i, j] <- c^(abs(i - j))
  }
}
library(mvtnorm)
set.seed(100)
X <- rmvnorm(n,matrix(0, p, 1), Sigma)
# generate beta and gamma
beta_true <- matrix(0, p, 1)
for( i in 1: k){
  beta_true[i, 1] = A
  if ( i%%3 == 1){
    beta_true[i, 1] = -A
  }
}
gamma_true <- D %%% beta_true
S0 <- which(gamma_true!=0)
# generate varepsilon
set.seed(1)
# generate noise and y
varepsilon <- rnorm(n) * sqrt(sigma)
y <- X %%% beta_true + varepsilon
filter_result <- sk.filter(X, D, y, option)
Z_path <- filter_result$Z
t_Z_path <- filter_result$t_Z

```

---

sk.select

*split knockoff selector given W statistics*


---

### Description

split knockoff selector given W statistics

**Usage**

```
sk.select(W, q, option)
```

**Arguments**

W statistics  $W_j$  for testing null hypothesis  
 q target FDR  
 option option\$method can be 'knockoff' or 'knockoff+'

**Value**

S array of selected variable indices

---

W\_sign

*W statistics generator for directional FDR control*

---

**Description**

generate the split knockoff statistics W for a split LASSO path, take the directional effect into account

**Usage**

```
W_sign(X, D, y, nu, option)
```

**Arguments**

X the design matrix  
 D the linear transform  
 y the response vector  
 nu the parameter for variable splitting  
 option options for creating the Knockoff statistics option\$eta specify the choice of eta for creating the knockoff copy; option\$lambda specify the choice of lambda for the split LASSO path

**Value**

W: the knockoff statistics  
 Z: feature significance  
 r: the sign estimator  
 t\_Z: knockoff significance

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W_support	<i>W statistics generator for FDR control</i>
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---

**Description**

generate the split knockoff statistics  $W$  for a split LASSO path, only consider the support set estimation

**Usage**

```
W_support(X, D, y, nu, option)
```

**Arguments**

X	the design matrix
D	the linear transform
y	the response vector
nu	the parameter for variable splitting
option	options for creating the Split Knockoff statistics; option\$eta specify the choice of eta for creating the knockoff copy; option\$lambda specify the choice of lambda for the split LASSO path

**Value**

W: the knockoff statistics  
Z: feature significance  
t\_Z: knockoff significance

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